Static Analysis of Laminated Hybrid Composite Plates using FSDT

D. Dhanunjaya Raju and V. V. Subba Rao

Abstract-- *Advanced Composite materials are replacing traditional metals in many engineering applications due to their attractive properties which are suitable for structural applications. Hybrid composites have greater flexibility to tailor the composites according the design requirements. In this paper an attempt is made to study the structural response of hybrid composite plates subjected to transverse load with variable ply angles and aspect ratios. A sub-laminate concept which satisfies the continuity conditions is used. Graphite/Epoxy and Kevlar/Epoxy sub-laminates are used to make the laminated composite plates. A systematic procedure is being developed to study the static behavior of hybrid composites using First Order Shear Deformation Theory (FSDT). The procedure is implemented through MATLAB programming. To validate the developed code, a simply supported laminated plate of specially orthotropic subjected to sinusoidal loading is considered. The numerical results, through developed code have been validated with the published results. The results are found good agreement with the published results. Then the developed code is further extended to the hybrid composite plates with different sub-lamination scheme. Numerical results are produced for different hybrid composite plates subjected to simply supported transverse sinusoidal loading. A parametric study has been carried out to find the best possible hybrid composite for the above loading.*

Keywords--*Hybrid Composites, FSDT, Advanced composites*

I. INTRODUCTION

Composite materials are replacing traditional metals in many engineering applications due to its light weight and high strength, due to these properties composite materials are used in structural engineering applications. Thus, an accurate stress analysis of the laminated composite plates is important in the design. To predict the stress distribution of composite plates, a number of plate theories have been proposed. In the classical laminate plate theory (CLPT), the transverse shear stresses are neglected[5]. However, in thick and moderately thick laminated plates the theory gives inaccurate results as there occur considerable shear deformations. So, it is obvious that transverse shear deformations have to be taken into account in the analysis. One of the well known plate theories are Reissner and Mindlin model E. Reissner [1], R. D. Mindlin [2]. In FSDT a shear correction factor is used to compensate for the assumed uniform transverse shear strain variations over the entire plate thickness. nondimensionalized deflections and stresses in laminated plate subjected to sinusoidal load using FSDT are given in [5]. In the present days a lot of research is going on for the design of hybrid laminated composites because it overcomes the drawbacks of traditional laminated composites, they are more stiffer, have high strength and ductility, improved fracture toughness, longer fatigue life and excellent impact resistance compared to traditional composite materials. This inspired us to do analysis on the laminated hybrid composite plates under mechanical loads.

II. FIRST ORDER SHEAR DEFORMATION THEORY - FSDT

First Order Shear Deformation Laminated Plate Theory (FSDT) is an extension of the Classical Lamination Plate theory. CLPT is based on kirchhoff assumptions in which transverse normal and shear stresses are neglected. These transverse shear stresses are derived by using 3D elasticity equilibrium equations these are not accurate but the equilibrium derived transverse stress are sufficiently accurate for homogeneous and thin plates. These are not accurate if the plates are relatively thick i.e (a/h<20) In FSDT transverse shear stresses are

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calculated and transverse normal stress is neglected. By using FSDT inter laminar shear stresses can be calculated through constructive equations. These constructive equations are simpler than equilibrium equations. CLPT is suitable for only thin plates but FSDT is suitable for relatively thick plates (a/h<20).

a. Simply supported symmetric Cross-Ply Laminated Plate

The mechanical loads are expanded in double Fourier sine series [5]

$$q(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} Q_{mn}\sin\alpha x\sin\beta y \tag{1}$$

It is a one –term solution ($Q_{mn}=q_0$ and $m=n=1$) for a sinusoidally distributed load, the solution is a closed form solution. For other types of loads, the Navier solution is a series solution, which can be evaluated for a sufficient number of terms in the series.

$$Q_{mn}=\frac{4}{a\,b}\int_0^b\int_0^a q(x,y)\sin\alpha x\sin\beta y\,dx\,dy \tag{2}$$

Where $\alpha=\frac{m\pi}{a}$ and $\beta=\frac{n\pi}{b}$

For specially orthotropic symmetric laminated plates from [5]

$$\begin{bmatrix} \hat{S}_{11} & \hat{S}_{12} & 0 & 0 & 0 \\ \hat{S}_{12} & \hat{S}_{22} & 0 & 0 & 0 \\ 0 & 0 & \hat{S}_{33} & \hat{S}_{34} & \hat{S}_{35} \\ 0 & 0 & \hat{S}_{34} & \hat{S}_{44} & \hat{S}_{45} \\ 0 & 0 & \hat{S}_{35} & \hat{S}_{45} & \hat{S}_{55} \end{bmatrix} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ X_{mn} \\ Y_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ Q_{mn} \\ 0 \\ 0 \end{Bmatrix} \tag{3}$$

Where

$$\begin{aligned} \hat{S}_{11}&=\left(A_{11}\alpha^2+A_{66}\beta^2\right), & \hat{S}_{12}&=\left(A_{12}+A_{66}\right)\alpha\beta \\ \hat{S}_{22}&=\left(A_{66}\alpha^2+A_{22}\beta^2\right), & \hat{S}_{34}&=K.A_{55}\alpha \\ \hat{S}_{44}&=\left(D_{11}\alpha^2+D_{66}\beta^2+KA_{55}\right), & \hat{S}_{35}&=K.A_{44}\beta \\ \hat{S}_{33}&=K\left(A_{55}\alpha^2+A_{44}\beta^2\right), & \hat{S}_{45}&=\left(D_{12}+D_{66}\right)\alpha\beta \\ \hat{S}_{55}&=\left(D_{66}\alpha^2+D_{22}\beta^2+KA_{44}\right) \end{aligned} \tag{4}$$

Where A=Extensional stiffness matrix, D=Bending Stiffness Matrix. [5]

Solution for the above (3) is given by [5]

$$w_0(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{mn}\sin\alpha x\sin\beta y \tag{5}$$

$$\phi_x(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} X_{mn}\cos\alpha x\sin\beta y \tag{6}$$

$$\phi_y(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} Y_{mn}\cos\alpha x\sin\beta y \tag{7}$$

Where

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$$W_{mn}=\frac{Q_{mn}}{b_{mn}} \quad ; \quad X_{mn}=\frac{b_1 Q_{mn}}{b_1 b_{mn}} \quad ; \quad Y_{mn}=\frac{b_2 Q_{mn}}{b_0 b_{mn}} \tag{8}$$

Where $b_{mn}=\hat{S}_{33}+\hat{S}_{34}\frac{b_1}{b_0}+\hat{S}_{35}\frac{b_2}{b_0}$; $b_0=\hat{S}_{44}\hat{S}_{55}-\hat{S}_{45}\hat{S}_{45}$

$$b_1=\hat{S}_{45}\hat{S}_{35}-\hat{S}_{34}\hat{S}_{55} \quad ; \quad b_2=\hat{S}_{34}\hat{S}_{45}-\hat{S}_{44}\hat{S}_{35} \tag{9}$$

The in–plane normal stresses (10)-(11), shear stress (12) and Transverse shear stresses (13)-(14) are expressed in the form as

$$\sigma_{xx}^{(k)}=-z\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left(\overline{Q}_{11}\alpha\ X_{mn}+\overline{Q}_{12}\beta Y_{mn}\right)\sin\alpha x\sin\beta y \tag{10}$$

$$\sigma_{yy}^{(k)}=-z\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left(\overline{Q}_{12}\alpha\ X_{mn}+\overline{Q}_{22}\beta Y_{mn}\right)\sin\alpha x\cos\beta y \tag{11}$$

$$\sigma_{xy}^{(k)}=-z\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left(-\overline{Q}_{66}\beta\ X_{mn}+\overline{Q}_{66}\alpha Y_{mn}\right)\cos\alpha x\cos\beta y \tag{12}$$

$$\sigma_{yz}^{(k)}=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left(\overline{Q}_{44}Y_{mn}+\overline{Q}_{44}\beta W_{mn}\right)\sin\alpha x\cos\beta y \tag{13}$$

$$\sigma_{xz}^{(k)}=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left(\overline{Q}_{55}X_{mn}+\overline{Q}_{44}\alpha W_{mn}\right)\cos\alpha x\cos\beta y \tag{14}$$

b. *Nondimensionalized Formulas*

Nondimensionalized maximum transverse deflection (15), normal stresses (16)-(17), shear stress (18) and Transverse shear stresses (19)-(20) are defined in the following equations is given as [5]

$$\overline{w}=w_0\left(\frac{a}{2},\frac{b}{2},\frac{h}{2}\right)\left(\frac{E_2 h^3}{a^4 q_0}\right)\times 10^2 \tag{15}$$

$$\overline{\sigma}_{xx}=\sigma_{xx}\left(\frac{a}{2},\frac{b}{2},\frac{h}{2}\right)\left(\frac{h^2}{a^2 q_0}\right) \tag{16}$$

$$\overline{\sigma}_{yy}=\sigma_{yy}\left(\frac{a}{2},\frac{b}{2},\frac{h}{4}\right)\left(\frac{h^2}{a^2 q_0}\right) \tag{17}$$

$$\overline{\sigma}_{xy}=\sigma_{xy}\left(a,b,-\frac{h}{2}\right)\left(\frac{h^2}{a^2 q_0}\right) \tag{18}$$

$$\overline{\sigma}_{xz}=\sigma_{xz}\left(a,b,-\frac{h}{2}\right)\left(\frac{h}{b q_0}\right) \tag{19}$$

$$\overline{\sigma}_{yz}=\sigma_{yz}\left(a,b,-\frac{h}{2}\right)\left(\frac{h}{b q_0}\right) \tag{20}$$

III. RESULTS

The MATLAB code has generated to validate the nondimensionalized transverse maximum central deflections, Normal stresses, Shear stress, Transverse Shear stresses of a simply supported square, cross-ply [0/90/90/0] composite plate subjected to sinusoidally distributed load [5].

The material properties are E1/E2=25, G12=G13=0.5E2, G23=0.2E2 and υ12=0.25.K=5/6

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Table 1. Effect of transverse shear deformation on nondimensionalized maximum transverse deflections and Normal stresses of Simply Supported symmetric cross-ply square plate

a/h	Reference [5] $\overline{w}$	Present $\overline{w}$	Reference [5] $\overline{\sigma}_{xx}$	Present $\overline{\sigma}_{xx}$	Reference [5] $\overline{\sigma}_{yy}$	Present $\overline{\sigma}_{yy}$
10	0.6627	0.6627	0.4989	0.4989	0.3614	0.3614
15	0.5367	0.5367	0.5192	0.5192	0.3144	0.3144
20	0.4912	0.4912	0.5273	0.5273	0.2957	0.2957
50	0.4409	0.4409	0.5368	0.5368	0.2737	0.2737
100	0.4337	0.4337	0.5382	0.5382	0.2705	0.2705

Table 2. Nondimensionalized maximum transverse deflections, shear stresses and Transverse shear stresses of simply Supported symmetric cross-ply square plate

a/h	Reference[5] $\overline{\sigma}_{xy}$	Present $\overline{\sigma}_{xy}$	Reference[5] $\overline{\sigma}_{xy}$	Present $\overline{\sigma}_{xy}$	Reference [5] $\overline{\sigma}_{yz}$	Present $\overline{\sigma}_{yz}$
10	0.0241	0.0241	0.4165	0.4165	0.1292	0.1292
15	0.0227	0.0227	0.4311	0.4311	0.1145	0.1145
20	0.0221	0.0221	0.4370	0.4370	0.1087	0.1087
50	0.0214	0.0214	0.4438	0.4438	0.1019	0.1019
100	0.0213	0.0213	0.4448	0.4448	0.1008	0.1008

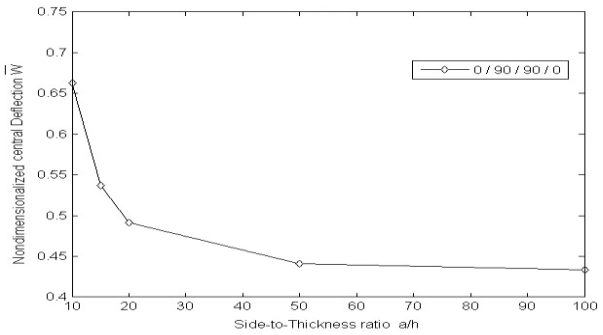


Fig.1 Nondimensionalized transverse deflections versus side to thickness ratio a/h

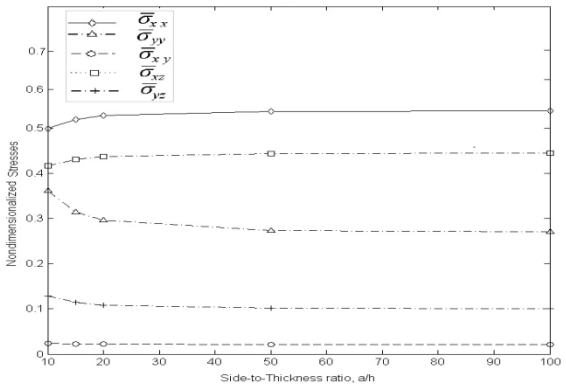


Fig.2 Nondimensionalized stresses versus side to thickness ratios a/h

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The numerical results, through developed code have been validated with the published results and are good agreement with the published results

Then the MATLAB code is further extended for finding out the deflections and stresses for hybrid composites of different lamination schemes.

The properties of Graphite/epoxy (G):[5] and Kevlar /epoxy (K):[7] are in (GPa)

(G) : E1=137.9, E2=8.96, G13=7.20, G23=6.21, G12=7.20, υ12=0.30

(K): E1=87, E2=5.5, G13=2.2, G23=2.2, G12=2.2, υ12 = 0.56

Table 3. Nondimensionalized maximum Transverse deflections, normal stresses, shear stresses and transverse shear stresses for Hybrid square laminated plates with a/h=10 subjected to SSL, with different fibre orientations

Orientation of fibers	Lamination Scheme	$\overline{w}$	$\overline{\sigma}_{xx}$	$\overline{\sigma}_{yy}$	$\overline{\sigma}_{xy}$	$\overline{\sigma}_{xz}$	$\overline{\sigma}_{yz}$
[0/90/90/0]	G/K/G/K	0.5673	0.3659	0.2125	0.0194	0.1426	0.0461
	K/G/G/K	0.6831	0.4578	0.4053	0.0239	0.1443	0.0497
	G/G/G/G	0.7097	0.4590	0.2600	0.0498	0.3079	0.1023
	G/K/K/G	0.7960	0.4812	0.1829	0.0537	0.4557	0.1447
	K/K/K/K	0.8387	0.4908	0.3062	0.0271	0.2903	0.0917
	K/G/K/G	0.9243	0.5777	0.3357	0.0636	0.4668	0.1509
[0/90/0/90]	G/K/G/K	0.5470	0.0312	0.1925	0.0186	0.1031	0.0846
	K/G/G/K	0.6629	0.0392	0.3707	0.0232	0.0944	0.0944
	G/G/G/G	0.6879	0.0392	0.2363	0.0481	0.2051	0.1769
	G/K/K/G	0.7626	0.0413	0.1593	0.0508	0.3088	0.2664
	K/K/K/K	0.7927	0.0427	0.2573	0.0253	0.1910	0.1910
	K/G/K/G	0.8910	0.0501	0.2951	0.0610	0.2769	0.2910
[0/45/45/0]	G/K/G/K	0.5324	0.3371	0.0627	0.0181	0.1437	0.0422
	K/G/G/K	0.6206	0.4075	0.4075	0.0215	0.1444	0.0443
	G/G/G/G	0.6669	0.4258	0.0829	0.0466	0.3075	0.0948
	G/K/K/G	0.7643	0.4538	0.0561	0.0512	0.4650	0.1343
	K/K/K/K	0.7912	0.4462	0.0908	0.0252	0.3009	0.0811
	K/G/K/G	0.8673	0.5323	0.1068	0.0592	0.4703	0.1382
[0/30/30/0]	G/K/G/K	0.5446	0.3450	0.0289	0.0186	0.1469	0.0372
	K/G/G/K	0.6399	0.4208	0.0596	0.0223	0.1480	0.0376

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G/G/G/G	0.6804	0.4348	0.0400	0.0477	0.3127	0.0849
G/K/K/G	0.7780	0.4612	0.0264	0.0523	0.4761	0.1219
K/K/K/K	0.8157	0.4587	0.0422	0.0262	0.3127	0.0693
K/G/K/G	0.8872	0.5448	0.0517	0.0609	0.4809	0.1219

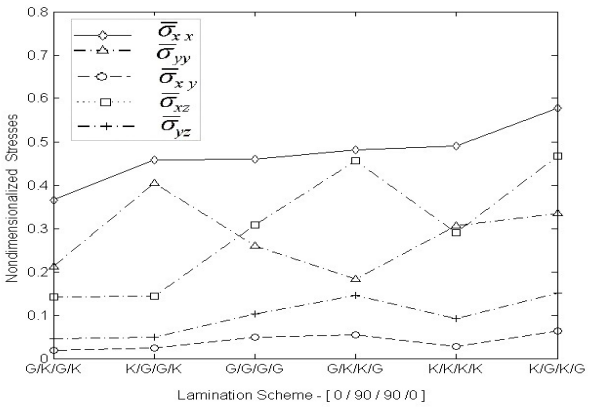


Fig.3 Nondimensionalized central Deflection versus different hybrid lamination schemes with different fiber orientations

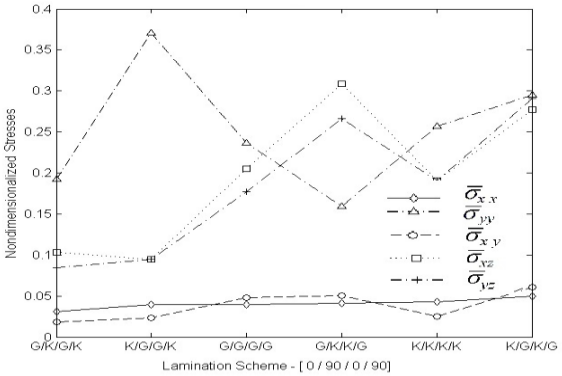


Fig.4 Nondimensionalized stresses versus different hybrid lamination schemes

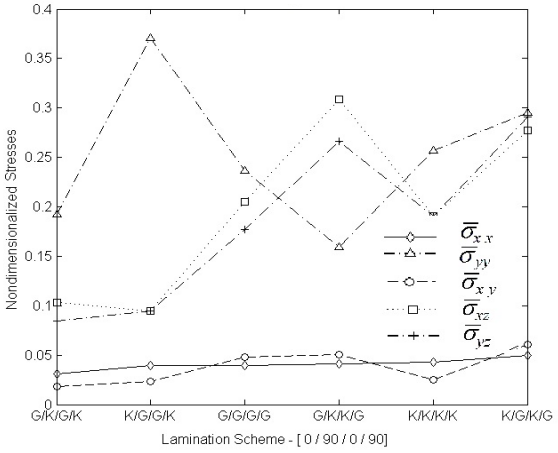


Fig.5 Nondimensionalized stresses versus different hybrid lamination schemes

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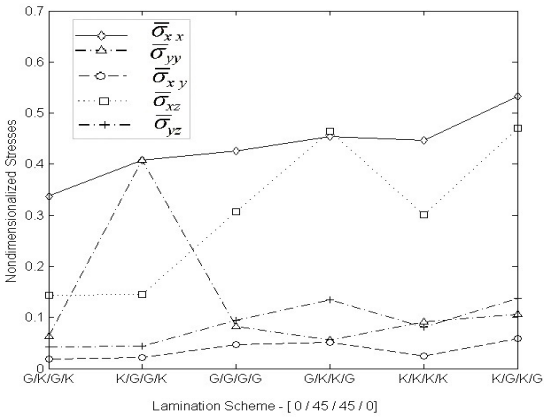


Fig.6 Nondimensionalized stresses versus different hybrid lamination schemes

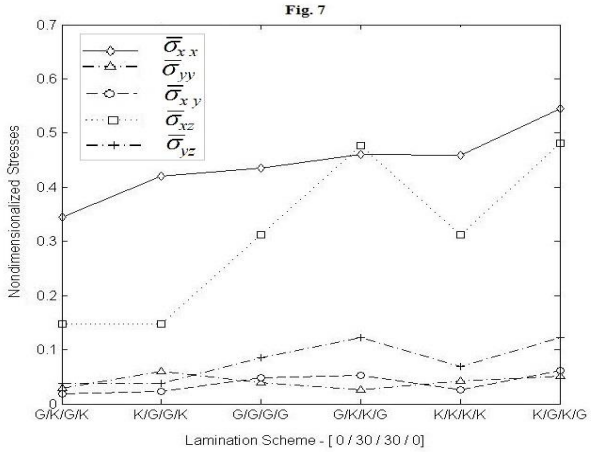


Fig.7 Nondimensionalized stresses versus different hybrid lamination schemes

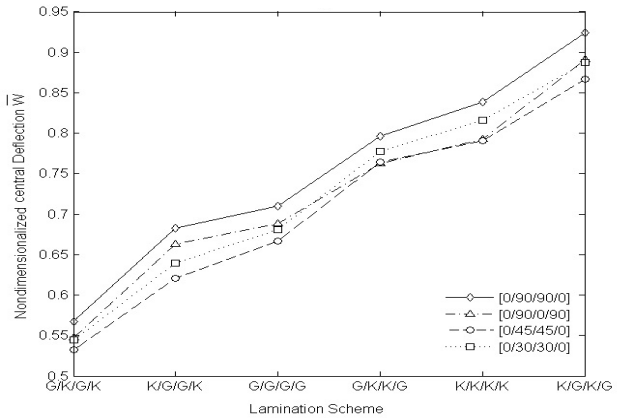


Fig. 8 Nondimensionalized stresses versus side -to thickness ratios a/h

IV. CONCLUSION

The developed MATLAB code is validated for the nondimensionalized transverse central deflections, Normal stresses, Shear stress, Transverse Shear stresses of cross-ply laminate [0/90/90/0]. The numerical results, through developed code have been validated with the published results. The results are found good agreement

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with the published results. Then the developed code is further extended to the hybrid composite plates with different sub-lamination scheme. Numerical results are produced for different hybrid composite plates subjected to simply supported transverse sinusoidal loading. For the same graphite/Epoxy and Kevlar/Epoxy laminas by changing the orientation of fibers [0/90/90/0] laminate produces higher and[0/45/45/0] laminate produces lower nondimensionalized deflections. By varying the hybrid lamination schemes [G/K/G/K] laminate produces lower and [K/G/K/G] laminate produces higher nondimensionalized deflections.

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