Controller Design for an Automatic Feedback Control System

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Abstract--- *Moment of inertia is an important property of the mechanical aspect of the robot and affects its dynamic performance. This paper presents the development of a control model in the transfer function form to minimize the moment of inertia of a stationary robotic system.*

Keywords--- *SCARA, Moment of Inertia, Control*

I. INTRODUCTION

THE property by virtue of which it resists any change in its state of rest or of uniform motion is called as inertia. Two types of inertia exist, viz., Translatory inertia and the rotary inertia. Translatory inertia is defined as the inertia that is offered by the translation of a body due to the application of a force and depends on the mass and acceleration ; i.e., F = ma. Translatory inertia is due to the mass [2]. Translatory inertia is called as Mass. Rotational inertia is defined as the inertia that is offered by the rotation of a body due to the application of torque and depends on moment of inertia and angular acceleration ; i.e., $T = I\alpha$. Rotational inertia is due to the MI. Rotational inertia is called as Moment of Inertia [1].

Three important concepts of any body or matter are the concepts of relative distribution of area, mass and the weight. Quantitative estimates of the relative distribution of the area and the mass over the regions of interest are made by the concept of Moments of Inertia [MI] of the object. The concept of inertia is provided by the Newton's laws of motion [4]. Moment of inertia plays a very important role in the dynamics and kinematics of robot mechanisms. A robot arm is made of links + joints and these joints are driven by actuators (motors / pistons) [3].

The arm dynamics of the robot is the spatial displacement or the movement of the robot in the 3D space with a particular velocity and acceleration. It mainly depends on the mass and MI. Moment of inertia depends on the positioning, velocity, acceleration, moments, torques, stresses, strains in links deflection effect, bending effect, buckling effect, bending stresses, vibrations of the links, shear stresses, torsional stresses & torsional strains related to the links of the robot arm.

MI of a part of a robot, say a link gives the information about the mass distribution of the part at a particular distance w.r.t. a particular axis. When the mass of the link increases, its weight increases, thereby MI increases, thereby increasing the torque rating and size of the motors. More power is required to drive the joints. The drive power amplifier required to drive the actuators would be of higher rating. To overcome this, joint - link movement motors are to be light in weight and the link weight has to be reduced (links should be light in weight, but robust, sturdy, should not bend). Hence, robots are designed in such a way that MI of the links is reduced, so that less torque is sufficient to move the links [5].

Note that MI is responsible for the creation of torque. The MI also depends on the axis. Say, for example, if the area of the link is more, then MI is more (link horizontal) and it will bend due to application of a force at the other end ($F_{bending}$ more), if the area of link is less (vertically placed), then, MI is less and it will not bend due to application of weight at the other end ($F_{bending}$ less). MI $\uparrow$, the T requirement to activate a joint increases in order to manipulate physical objects.

The robot manipulation depends on the MI and the mass, weight of its components such as links, motors, grippers, tools, payload weight etc., in addition, the torque developed by the motor depends on the load which in turn depends on the MI which in turn also depends on the column effect / buckling effect in the links, bending stresses and shear stresses that are developed in the links when they are loaded at one end. All these factors have to be considered for robot stability, i.e., the link design totally depends on the MI [6].

The spatial displacement of the links should be according to the speed profile curve. So, links should be as light as possible and made up of high grade aluminum, the joint motors should be light or instead of mounting the heavy parts at the joints, mount it at the base and transmit the power from the base to the joint by making use of chains, ropes and pulleys. The joint-link pair mechanism should be designed with less weight so that the robot control will become easy and effect of MI on the links is reduced and less torque is required by the motors to displace the load.

The heavy objects such as the motors and the gear mechanisms have to be mounted at the base and power should be transmitted from the base to the respective joints via the transmission devices (the chains, gears, ropes, pulleys, etc.) like in Rhino XR-3 robot arm. This will

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increase the positional accuracy of the robot and reduce the overall MI of the robot [7].

The length of the arm should go on reducing as one moves radially away from the robot base, i.e., the deflection ratio should be maintained (Ex. SCARA robot, a_1 = 425 mm, a_2 = 375 mm). If a third link would have been, then the length of that link would be < a_2. The main factors which are affecting the control problems and the MI are the Positioning, Velocity, Acceleration, Moments, Torques, Stresses, Strains in links, Deflection effect, Bending effect, Buckling effect, Bending stresses, Vibrations of the links, Shear stresses, Torsional stresses and strains related to the links of the robot arm and the controller rating. All these factors have to be considered while designing the robot and its control strategy for its successful robotic manipulation [8].

The paper is organized as follows. A brief introduction about the MI concepts etc., and its effects was discussed in the previous paragraphs. Section 2 gives a small review of the physical construction of the robotic system that was designed and fabricated. Section 3 gives a overview of the concepts in the design of the controller to overcome the MI. In section 4, for the physical model developed in the section 2, a control model in the mathematical form along with the experimental results are presented. Finally, the conclusions are presented in the last followed by the references.

II. INTRODUCTION TO THE ROBOT DESIGN

The designed robot for which we developed the control model is a 3 DOF stationary robot arm having base, elbow, vertical extension and consisting of both rotary and prismatic joints. There is no tool yaw and tool pitch (only tool roll) [1]. There are 3 joints, 3 axis (2 major axes - base, elbow, and one minor axis vertical extension) [9]. The 3 DOF's are given by Base, Elbow, Vertical Extension as shown in the Fig. 2, i.e., there are 3 rotary joints and 1 prismatic joint [9]. Since n = 3 ; 12 KP's are to be obtained and 4 RHOCF's are to be attached to the various joints [2] as shown in the Link Coordinate Diagram (LCD) in Fig. 3 [10].

The vector of joint variables is a combination of θ and d,

$$\text{i.e., } q = \{\theta , d\}^T.$$

Vector of joint variables are

$$q = \{\theta_1 , \theta_2 , d_3 \}^T.$$

Vector of joint distances are

$$d = \{ d_1 , 0 , d_3 \}^T = \{500 , 0 , d_3\}^T \text{ mm} .$$

Vector of link lengths are

$$a = \{a_1, a_2 , 0 \}^T = \{300 , 250 , 0\}^T \text{ mm}.$$

Vector of link twist angles are

$$\alpha = \{\alpha_1, \alpha_2, 0 \}^T = \{\pm \pi , 0 , 0 \}^T.$$

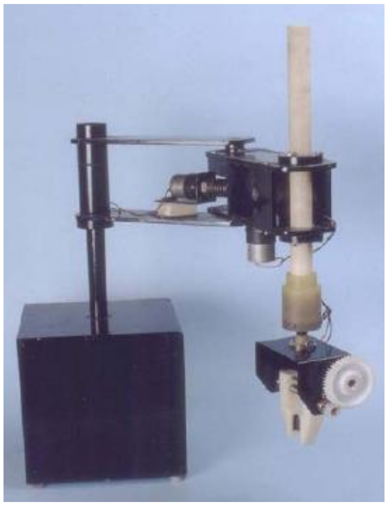

Figure 1: Designed and Fabricated SCARA Robotic arm Unit

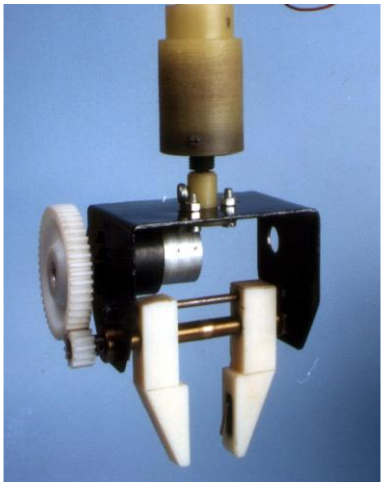

Figure 2: Robot's Gripper

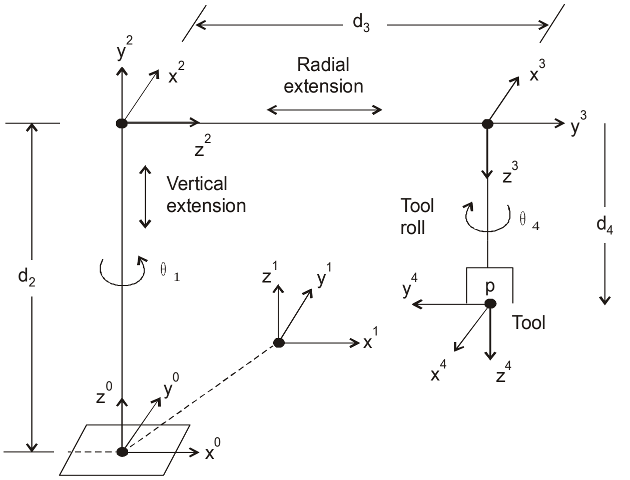

Figure 3: The Link Coordinate Diagram of the Robot

All the 3 joint axes are vertical in nature (all the z - axes can be pointing down or up) as shown in Fig. 3. The 3 (B, E, VE) axes are called as the major axes and are used for positioning the wrist and the last one is used to orient the tool or the gripper in the direction of the object [10]. The first 3 axes determine the shape and size of work envelope [8]. It consists of a L shaped structure, to the end of which the second link is attached. There are 2 links a_1 and a_2 which move parallel to work surface. The vertical extension d_3 is variable and moves in a direction $\perp^r$ to the work surface, length of the gripper / tool / EE is d_3 and its maximum protrusion is 300 mm [2].

The tool / gripper / EE is permanently pointing down as shown in Fig. 2 and can rotate in a plane $\perp^r$ to the work surface plane $x^0\ y^0$. The approach vector r^3 is fixed, i.e., $r^3 \perp^r x^0\ y^0$ (work surface) plane ; $r^3 = - z^0$. Because of this reason, our designed and dynamically modeled SCARA robot can do robotic manipulation directly from above the object when exact perpendicularity is required [11]. The SCARA robot is a minimal representation of any robot [3]. Our SCARA robot is a special type of polar / spherical coordinate robot in which the major axes are R R P [7].

III. CONTROLLER CONCEPTS

A controller is a device which controls each and every operation that is taken place inside the robotic system making decisions. The functions of a robot controller are : The robot is a mechanical system that must be controlled in order to accomplish a useful task (say , a PNP operation). The task involves the movement of the manipulator arm from the source to the destination, so that the primary function of the robot control system is to position and orient the wrist with a specific speed, precision, accuracy. The controller serves as a interface between the robot and the computer.

It controls each and every operation of the robot making decisions [11]. The controller consists of a driver unit, sometimes a power supply unit, electronic logic circuits, feedback circuitry, sensing circuitry, few amplifiers, IC chips and the actuators. The task of the control system is to execute the movements and actions of the manipulator according to the robot program and to coordinate the interaction with the environment. Controllers can operate either in open loop or in closed loop. Each axis of motion of the robot arm is separately actuated by a control circuitry which contains a driver unit which converts the electrical command signals of the computer to mechanical motions.

Drives for computerized robotic systems designed are of electric type [12]. The purpose of the controller is to compare the actual output of the plant with the input command and to provide a control signal which will reduce the error to zero or as close to zero as possible. The controller generally consists of a summing junction where the input and the output signals are compared, a control device which determines the control action, the necessary power amplifiers and the associated hardware devices to accomplish the control action in the plant [13].

Robot control problem means how to control it to do a particular task effectively in its work space. Robot control problem is taken care of by a controller which is used to interface between the robot and the computing system. Various types of controllers such as P, PI, PD, PID, FOS, POF, H_2, H_∞, Sliding mode controller, programmable logic controllers, microcontrollers can be designed to overcome the MI & to do a correct task [14]. The designed controller is shown in the form of a block diagram in the Fig. 4.

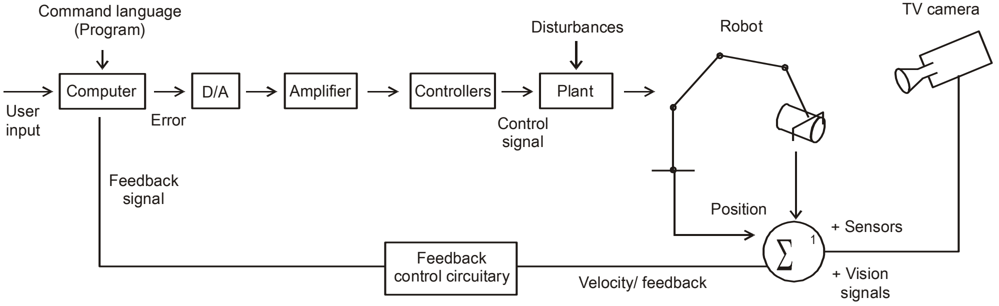

Figure 4: Block Diagram of the Controller

IV. CONTROLLER DESIGN

Consider an electromechanical servo system in the designed robot, where the mechanical drive unit consists of an arm controlled DC servomotor and a gear train of high gear reduction ratio 'n' as shown in Fig. 5 [22]. The block diagram is in the Fig. 6. It gives the position and velocity feedback of the servo system. The position input signal θ_p radians acting on the system (electrical input to actuator - voltage, v) results in an output signal θ_0 radians (mechanical output: - angular displacement,θ). The servos used for robotic control application is a Type-1 system, i.e., it consists of 1 integrator in the feedback loop [15].

⇒	Forward path
→	Feedback path
K_e	Motor back e.m.f. constant
K_m	Motor torque constant
k_p	Position feedback gain constant
k_v	Velocity feedback gain constant
A	Amplifier gain
J_m , J_L	Rotor moment inertia and load MI
N	Gear reduction ratio
D	Viscous friction coefficient
θ_0	Controlled output
θ_p	Reference input

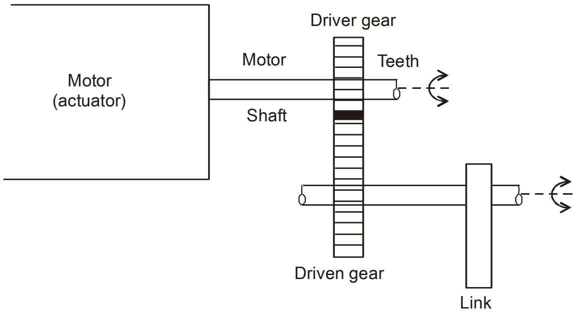

Figure 5: Gear Reduction Unit

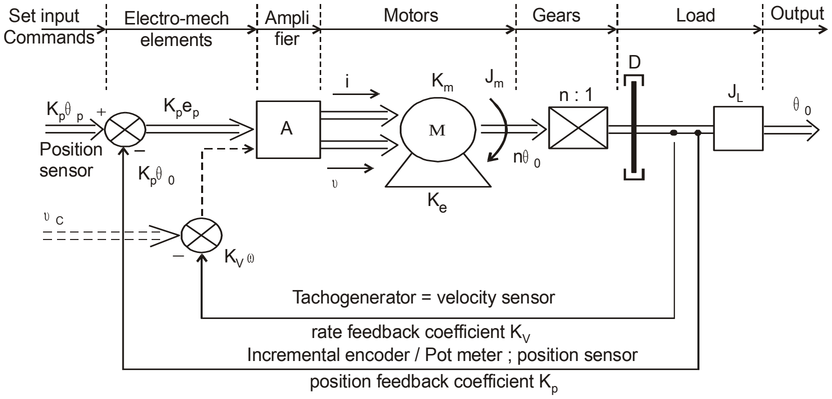

Figure 6: Schematic Block-Diagram of a Joint-Link Robot Servo System Actuated by a DC Servo Motor Via a Gear Train

Two feedback loops are designed for the controller for the robotic unit [33], [34]. They are as follows:

- Output θ_0 is fedback using position sensor and the difference between the input and the output which is proportional to the position error signal $e_p(v)$ is found out using equation [16]

$$e_p = K_p(\theta_p - \theta_0), \qquad (1)$$

where, K_p is position feedback gain coefficient (V / rads)

- Output θ0 is fedback using a tachometer device which gives a signal ev(V) which is proportional to the velocity of the output shaft rotation and is given by

$$e_v = -K_v \frac{d\theta_0}{dt} \qquad (2)$$

Where, K_v is velocity gain coefficient (V / rad . sec)

The two feedback signals given by Eqns. (1) and (2) are superimposed using a amplifier to give an effective error signal resulting in current control strategy for driving the robot actuators and is given by Eq. (3) as

$$i \propto^{al} (e_p - e_v)$$

$$i = A\left[K_p \ \theta_p - \theta_0 \ - K_v \frac{d\theta_0}{dt}\right] \qquad (3)$$

Where A is the gain of the amplifier and is the proportionality constant. The gross torque (total torque) developed by servomotor on its shaft to drive the joint is directly proportional to the current and is given by Eq. (4).

$$T_g \propto^{al} i$$

$$T_g = k_m \ i$$

$$T_g = A \ K_m\left[K_p \ \theta_p - \theta_0 \ - K_v \frac{d\theta_0}{dt}\right] \qquad (4)$$

Where, k_m is the motor gain constant.

Thus, we have explained how the control torque is generated [32]. Now, we shall use this generated torque to control the individual joint-link arm segments. A substantial part of the generated torque is absorbed in accelerating the rotor MI, J_m (K_g m^2) which also includes the inertia of the gear train on the rotor shaft ; i.e., some part of the total torque produced is wasted in over coming the losses in the servomotor (inertia losses, damping losses, frictional losses, windage losses, copper losses, hysteresis and eddy current losses, mechanical losses) [17]. Consider at this stage the only load to be the MI, J_m. Then, the torque T_m in N-m used to accelerate this MI is given by Eq. (5). The motor torque is given by [18]

$$T_m = J_m \frac{d^2 \ n\theta_0}{dt^2} = n \, J_m \frac{d^2 \ \theta_0}{dt^2} \qquad (5)$$

Subtract T_m from the gross torque T_g to give the rotor torque T_r which is used to control all the loads on motor shaft and is given by the Eq. (6).

∴, the useful torque available at the shaft of the rotor for driving the joint is = the gross torque – the wastage torque [19].

$$T_r = \left[T_g - T_m\right]$$

$$= T_g - n \ J_m \frac{d^2\theta_0}{dt^2} \qquad (6)$$

This torque T_r acts on the shaft of the servomotor. Therefore, its action must be reflected in slow running shaft of the gear train. The gear reduction unit magnifies the torque by gear ratio ‘n’ [27]. The link is connected to the motor shaft via a series of gear trains.

It is done so in order to convert the high speed rotation of the motor shaft into the slow speed running of the joint-link axis [20]. Since, the link cannot rotate at such a high speed, the speed of the motor is reduced and torque is magnified using gears (gears are nothing but speed reduction devices and torque magnification devices) [28]. Therefore, the useful torque on slow rotation shaft after gear train is given by the Eq. (7) as

$$n\,T_r = n\left[T_g - n\,J_m\frac{d^2\theta_0}{dt^2}\right] \qquad (7)$$

The load on the slow running shaft consists of

- Inertia’s of secondary part of gear unit.
- Effective inertia of arm segments.
- Gripper and tool.
- Pay load / part / object [29].

Combine all of them , the overall effective moment of inertia J′ is obtained and is reflected to the end of the arm segment [25].

Useful torque , $T = nT_r$ is used in accelerating the effective overall inertia J′ on the output shaft of load and is given by Eq. (8)

$$T' = J'\frac{d^2\theta_0}{dt^2}B \qquad (8)$$

Also, part of the useful torque T′ is used to overcome the viscous friction (proportional to velocity), coulomb friction (constant), stiction (instantaneous) [30].

∴, mechanical damping torque with viscous damping factor D is given by Eq. (9).

$$T_D = D\frac{d\theta_0}{dt} \qquad (9)$$

Where D is the viscous damping factor [21].

Required torque for driving the effective load and the useful torque generated by the motor are equal.

Equate them, we get a differential equation which gives the motion of the robot joint - link arm segments and is given by Eq. (10)

$$J'\frac{d^2\theta_0}{dt^2} + D\frac{d\theta_0}{dt} = n\left(T_g - n\,J_m\frac{d^2\theta_0}{dt^2}\right) \qquad (10)$$

Substituting Eq. (4) in Eq. (10), we get,

$$J'\frac{d^2\theta_0}{dt^2} + D\frac{d\theta_0}{dt} = n\left(A\,K_m\left[K_p\ \theta_p - \theta_0\ - K_v\frac{d\theta_0}{dt}\right] - n\,J_m\frac{d^2\theta_0}{dt^2}\right) \qquad (11)$$

Rearrange this Eq. (11) to get a second order linear differential equation.

The equation of motion of the controller is finally given by Eq. (12) as [31]

$$J'\frac{d^2\theta_0}{dt^2} + \left(\frac{D+AK_m nK_v}{n^2 J''}\right)\frac{d\theta_0}{dt} + \left(\frac{AK_m nK_p}{n^2 J''}\right)\theta_0 = \left(\frac{AK_m nK_p}{n^2 J''}\right)\theta_p \qquad (12)$$

Take the Laplace Transform (LT) of the above second order linear differential equation and get the transfer function of the controller as [23]

$$\left(\frac{D+AK_m nK_v}{n^2 J''}\right)s^2\theta_0(s) + \left(\frac{AK_m nK_p}{n^2 J''}\right)\theta_0(s) = \left(\frac{AK_m nK_p}{n^2 J''}\right)\theta_p(s) \qquad (13)$$

The Transfer Function [TF] of the robot control servo system is given by Eq. (14) and is shown in the form of a block-diagram in the Fig. 7.

$$G(s) = \frac{\theta_0(s)}{\theta_p(s)} = \frac{A\,K_m n\,K_p}{s^2 n^2 J'' + \ \ D + A\,K_m n\,K_v\ \ s + A\,K_m n\,K_p} \qquad (14)$$

This TF of the robot control system developed is of the second order form given by Eq. (15) as

$$G\,(s) = \frac{\omega_n^2}{s^2 + 2\delta\,\omega_n s + \omega_n^2} \qquad (15)$$

Where

ω_n= Undamped natural frequency of the system or natural frequency of the free or sustained oscillations,

δ = Damping factor (friction),

s = Complex variable consisting of a real and imaginary part = ($\sigma + j\,\omega$).

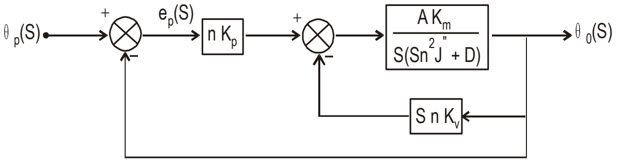

Figure 7: Block Diagram and Transfer Function Model of the Servo-Controlled Joint-Link Arm Segment of the Robot Arm

Comparing Eqns. (14) and (15), we get

$$\omega_n = \sqrt{\frac{A\,K_m n\,K_p}{n^2 J''}} \qquad (16)$$

$$\delta = \frac{D + A\,K_m n\,K_v}{\sqrt{A\,K_m n\,K_p n^2 J''}} \qquad (17)$$

The robot control system is designed for damping factor in the range $0.4 < \delta < 0.9$, as a result of which the output always exhibits damped oscillations (under damped response) and hence, system will be stable [26]. The problem regarding oscillation is that the structural natural frequencies of the joint-link arm segments vary inversely with the square root of the effective inertias between their minimum and maximum values [22].

The minimum and maximum values of the inertias are as follows : i.e., J''_{min} in no-load-no-stretch and J''_{max} in full-load-full-stretch positions. Both of these limiting values of inertia are obtainable from measurement . In practice , the variation of the ratio of the joint-link inertias $\frac{J''_{max}}{J''_{min}}$ within the robot workspace may vary by the ratio $10 : 1$. The experimental result of one of the joint of the robot arm when a particular value of torque given is shown below in Fig. 8 [24].

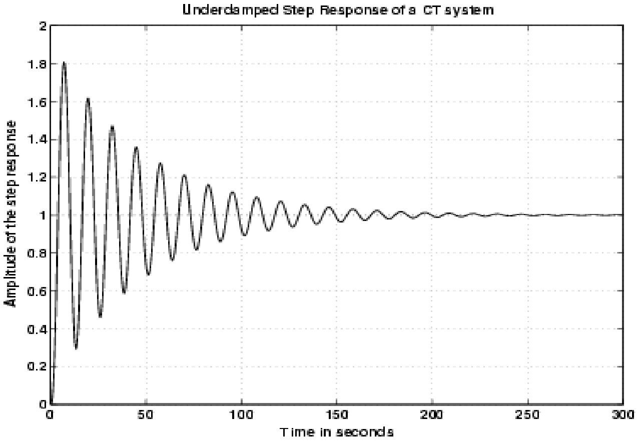

Figure 8: Experimental Result of a Particular Joint of the Robot System

V. CONCLUSIONS

A novel design of a mathematical control model was developed for a stationary robotic system. The control model was designed taking into account all the necessary damping, feedback, backlash, hysteresis, etc.,. The successful testing was also done. The controller developed was of the second order form.

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